FOSTER AND CAUTER SYNTHESIS USED IN DESIGNING ONE-PORTS RC

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Abstract – The paper, mainly intended for the didactic activity, presents the synthesis design of the RC type one-ports. Determination of the impedance function, followed first by its development in elementary fractions related to the poles, allows the obtaining of the Foster I and Foster II circuits, and then its development in continuous fractions allows to obtain the Cauer I and Cauer II circuits. Because synthesis is performed using the same impedance function, the obtained circuits are equivalent. This assertion is also supported by obtaining the same frequency characteristics across the entire frequency range in which circuits analysis is performed using the Orcad software.

1. INTRODUCTION

The work presents the synthesis-based design of the RC one-ports. The determination of the impedance function (real positive function) allows the performance of the four types of syntheses: Foster I and Foster II, Cauer I and Cauer II. The syntheses lead to four equivalent RC one-ports. The syntheses carried out correctly followed by the simulation with the help of Orcad programme, lead to four frequency characteristics, identical throughout the frequency domain, in which the analysis is being carried out.

The utilization of Mathcad programme allows the illustration, in logarithmical scale, of the real and ideal frequency characteristics of the impedance function determined.

2. DETERMINATION OF THE IMPEDANCE FUNCTION

The determination of the impedance function is performed knowing that it has the highest integer negative numbers as critical frequencies and $\lim_{s\to\infty} Z(s) = 1$.

The 2nd order functions Z_{RC}, having as critical frequencies the highest integer and negative numbers (taking into account the necessary alternation of the poly-zeros and the nature of the lowest critical frequency, which has to be a pole of the function) are as follows, [1]:

$$Z_1(s) = k_1 \cdot \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

and $$Z_2(s) = k_1 \cdot \frac{(s+2)}{(s+1)(s+3)}$$

Because:

$$\lim_{s\to\infty} Z_2(s) = 0$$

it results that only $Z_1(s)$ can meet the second condition as well, for $k_1 = 1$.

Therefore, the impedance function which has to be synthesized is:

$$Z(s)=\frac{(s+2)(s+4)}{(s+1)(s+3)} \quad (1)$$

3. FOSTER AND CAUER SYNTHESES

1. *Foster I Synthesis*

Foster I Synthesis of the RC one-ports consists in developing $Z(s)$ in elementary fractions, achieving each development term through an elementary one-ports and connecting them in series, [3], [5]:

$$Z(s)=k_\infty+\frac{k_1}{s+1}+\frac{k_2}{s+3} \quad (2)$$

Calculating, we obtain:

$$k_\infty=\lim_{s\to\infty} Z(s)=1 \quad (3)$$

$$k_1=\lim_{s\to -1}(s+1)\cdot Z(s)=\frac{3}{2} \quad (4)$$

$$k_2=\lim_{s\to -3}(s+3)\cdot Z(s)=\frac{1}{2} \quad (5)$$

Finally, we obtain:

$$Z(s)=1+\frac{1}{\frac{2s}{3}+\frac{2}{3}}+\frac{1}{2s+6} \quad (6)$$

The diagram of Foster I-type RC one-port, according to the development given by relation (6), is presented in figure 1.

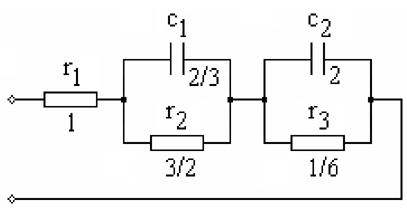

Figure 1. Foster I-type RC one-ports

Observation: By carrying out the synthesis, the corresponding circuit will be formed of dimensionless norm circuit elements $(r_k,\ c_k)$.

We obtain:

$$r_1=1; \quad r_2=\frac{3}{2}; \quad r_3=\frac{1}{6}; \quad c_1=\frac{2}{3}; \quad c_2=2 \quad (7)$$

The return to the non-norm values (R_k, C_k) is carried out based on the relations:

$$r_k=\frac{R_k}{R_u}; \quad R_k=r_k\cdot R_u; \quad c_k=\omega_u R_u C_k; \quad C_k=\frac{c_k}{\omega_u R_u} \quad (8)$$

For the synthesis of the one-ports, we consider as unit values of the elements:

$$\omega_u=1\,\text{Mrad/s}; \quad R_u=1\,\text{K}\Omega; \quad C_u=1\,\text{nF}$$

Calculating, we obtain the non-norm values:

$$R_1=1\text{K}\Omega; \quad C_1=0.667\text{nF}; \quad R_2=1.5\text{K}\Omega; \quad C_2=2\text{nF}; \quad R_3=0.166\text{K}\Omega \quad (9)$$

2. *Foster II Synthesis*

Foster II Synthesis of the RC one-port consists in developing $Y(s)/s$ in elementary fractions, clarifying $Y(s)$ in this development, achieving each fraction of $Y(s)$ through elementary one-ports and connecting them in parallel [3], [5].

We obtain:

$$\frac{Y(s)}{s}=\frac{k_0}{s}+\frac{k_1}{s+2}+\frac{k_2}{s+4} \quad (10)$$

Calculating, we obtain:

$$k_0=\lim_{s\to 0} s\cdot\frac{Y(s)}{s}=\frac{3}{8} \quad (11)$$

$$k_1=\lim_{s\to -2}(s+2)\cdot\frac{Y(s)}{s}=\frac{1}{4} \quad (12)$$

$$k_2=\lim_{s\to -4}(s+4)\cdot\frac{Y(s)}{s}=\frac{3}{8} \quad (13)$$

Finally, we obtain:

$$\frac{Y(s)}{s} = \frac{3}{8s} + \frac{s}{4(s+2)} + \frac{3}{8(s+4)} \quad (14)$$

and: $$Y(s) = \frac{3}{8} + \frac{1}{4+\frac{8}{s}} + \frac{1}{\frac{8}{3}+\frac{32}{3s}} \quad (15)$$

The diagram of Foster II-type RC one-port, according to the development given by relation (15), is presented in figure 2.

We obtain:

$$r_4 = \frac{8}{3}; \quad r_5 = 4; \quad r_6 = \frac{8}{3}; \quad c_3 = \frac{3}{8}; \quad c_4 = \frac{3}{32} \quad (16)$$

The return to the non-norm values (R_k, C_k) is performed based on the relations (8).

Calculating, we obtain the non-norm values:

$$R_4 = 2.667K\Omega; \quad C_3 = 0.125nF; \quad R_5 = 4K\Omega; \quad C_4 = 0.0937nF; \quad R_6 = 2.667K\Omega \quad (17)$$

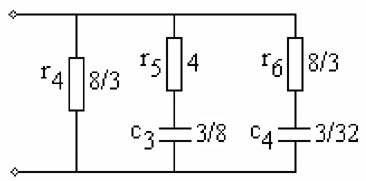

Figure 2. Foster II-type RC one-ports

3. *Cauer I Synthesis*

When the degrees differ by one unit, the synthesis in Cauer I scale of the RC real positive functions does not pose any problems, the development beginning by dividing the higher degree polynomial by the lower degree polynomial and then continuing the development algorithm in continuous fraction.

For equal degrees, in the first stage we will extract a constant (a resistance), then in the 2nd stage we will extract the pole from the infinity.

Since only Y_{RC} (not Z_{RC}) can have pole at the infinity, it results that the function in the 2nd stage of the algorithm is an admittance, therefore in the first stage it has to be an impedance. Thus, Cauer I development applies to $Z(s)$ in this case, obtaining the continuous fraction [3], [5]:

$$Z(s) = \frac{s^2+6s+8}{s^2+4s+3} \quad (18)$$

$$\frac{s^2+6s+8}{s^2+4s+3} = 1 + \cfrac{1}{\frac{s}{2} + \cfrac{1}{\frac{4}{3} + \cfrac{1}{\frac{3}{2}s + \cfrac{1}{\frac{1}{3}}}}} \quad (19)$$

The diagram of Cauer I-type RC one-port, according to the development (19), is presented in figure 3:

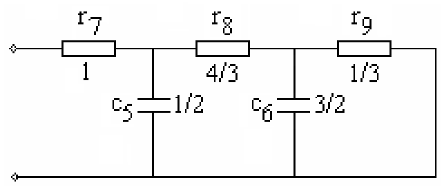

Figure 3. Cauer I-type RC one-ports

We obtain:

$$r_7 = 1; \quad r_8 = \frac{4}{3}; \quad r_9 = \frac{1}{3}; \quad c_5 = \frac{1}{2}; \quad c_6 = \frac{3}{2} \quad (20)$$

The return to the non-norm values (R_k, C_k) is carried out based on the relations (8).

Calculating, we obtain the non-norm values:

$$R_7 = 1K\Omega; \quad C_5 = 0.5nF; \quad R_8 = 1.334K\Omega; \quad C_6 = 1.5nF; \quad R_9 = 0.334K\Omega \quad (21)$$

4. *Cauer II Synthesis*

It does not pose any problems when the RC real positive function presents pole or zero in the origin, the algorithm beginning by dividing the polynomials with free term by the polynomial without a free term, obviously the polynomials are in ascending order.

When the function does not have pole in the origin, in the first stage we will extract a constant, then in the 2nd stage we will extract the pole from the origin. Since only Z_{RC} (not Y_{RC}) can have pole in the origin, it results that the function in the second stage of the algorithm is an impedance, therefore in the first stage it has to be an admittance [3], [5].

Therefore, Cauer II development applies to $Y(s)$ in this case, obtaining the continuous fraction:

$$Y(s)=\frac{3+4s+s^2}{8+6s+s^2} \qquad (22)$$

$$\frac{3+4s+s^2}{8+6s+s^2}=\frac{3}{8}+\cfrac{1}{\cfrac{32}{7s}+\cfrac{1}{\cfrac{49}{88}+\cfrac{1}{\cfrac{968}{2s}+\cfrac{1}{\cfrac{3}{44}}}}} \qquad (23)$$

The diagram for Cauer II-type RC one-port, according to the development (23), is presented in figure 4:

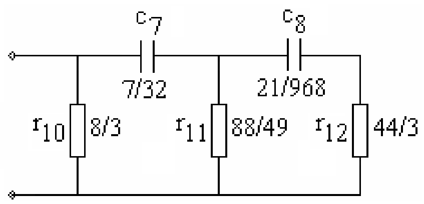

Figure 4. Cauer II-type RC one-ports

We obtain:

$$r_{10}=\frac{8}{3};\quad r_{11}=\frac{88}{49};\quad r_{12}=\frac{44}{3};$$
$$c_7=\frac{7}{32};\quad c_8=\frac{21}{968} \qquad (24)$$

The return to the non-norm values (R_k, C_k) is carried out based on the relations (8).

Calculating, we obtain the non-norm values:

$$R_{10}=2.667K\Omega;\quad C_7=0.218nF;$$
$$R_{11}=1.795K\Omega;\quad C_8=0.0216nF; \qquad (25)$$
$$R_{12}=14.667K\Omega$$

4. EXPERIMENTS

The illustration of the real and ideal frequency characteristics of the impedance function determined by relation (1), using Mathcad programme, is presented in figure 5, [3], [5].

$$\omega := 0.01, 0.1 .. 100$$

$$\omega 1 := 1 \qquad \omega 2 := 2$$

$$\omega 3 := 3 \qquad \omega 4 := 4$$

$$a1(\omega) := -20\cdot\log\left(\frac{\omega}{\omega 1}\right)\cdot\Phi(\omega-\omega 1)$$

$$a2(\omega) := 20\cdot\log\left(\frac{\omega}{\omega 2}\right)\cdot\Phi(\omega-\omega 2)$$

$$a3(\omega) := -20\cdot\log\left(\frac{\omega}{\omega 3}\right)\cdot\Phi(\omega-\omega 3)$$

$$a4(\omega) := 20\cdot\log\left(\frac{\omega}{\omega 4}\right)\cdot\Phi(\omega-\omega 4)$$

$$ar1(\omega) := -10\cdot\log\left(1+\frac{\omega^2}{\omega 1^2}\right)$$

$$ar2(\omega) := 10\cdot\log\left(1+\frac{\omega^2}{\omega 2^2}\right)$$

$$ar3(\omega) := -10\cdot\log\left(1+\frac{\omega^2}{\omega 3^2}\right)$$

$$ar4(\omega) := 10\cdot\log\left(1+\frac{\omega^2}{\omega 4^2}\right)$$

$$a(\omega) := a1(\omega)+a2(\omega)+a3(\omega)+a4(\omega)$$

$$ar(\omega) := ar1(\omega)+ar2(\omega)+ar3(\omega)+ar4(\omega)$$

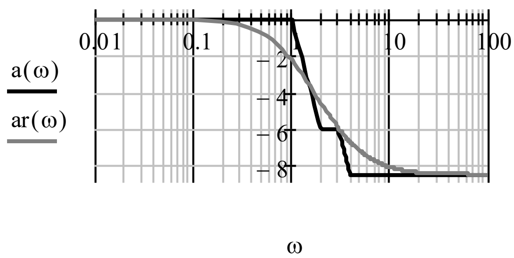

Figure 5. The real and ideal frequency characteristics of the impedance function

To simulate Foster and Cauer-type RC one-ports we utilize Orcad programme.

In figure 6 we present the circuits of Foster and Cauer-type RC one-ports, simulated in Orcad.

The real frequency characteristics obtained by simulating the circuits of Foster and Cauer-type RC one-ports in Orcad are presented in figure 7.

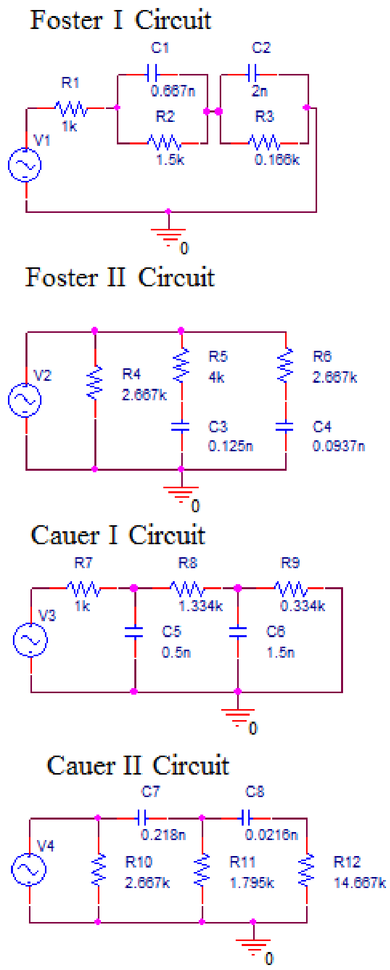

Figure 6. Circuits of Foster and Cauer-type one-ports

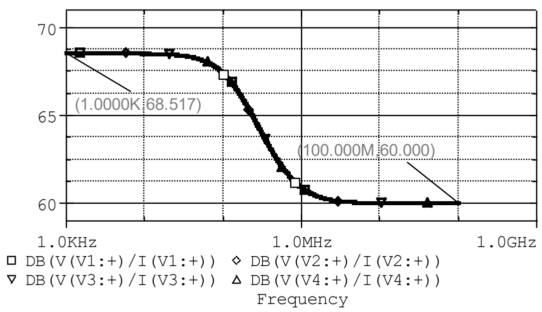

Figure 7. Real frequency characteristics

5. CONCLUSIONS

The correct determination of the impedance function, followed by the correct performance of the Foster and Cauer syntheses lead to the achievement of equivalent RC one-ports circuits.

The correct simulation of these circuits in Orcad allows the achievement of real frequency characteristics, identical throughout the frequency domain, in which the analysis is being carried out.

6. REFERENCES

[1] A. Mateescu, A. Şerbănescu, N. Dumitriu, L. Anton, L. Stanciu, G. Alexandrescu, "*Signals, Circuits and Systems*", ISBN 973-32-0495-1, Military Publishing House, Bucharest, 1998

[2] Rodica-Mihaela Teodorescu, Monica-Anca Chita, "*RC uniport analysis using virtual instrumentation*", Sibiu Alma Mater University National Conference with International Participation, Section 4, Engineering sciences, Fourth edition 25th – 27th of March, 2010.

[3] Teodorescu, Rodica Mihaela, "*Analysis and synthesis of circuits by engineering programming environments* ", ISBN 978-606-560-339-4, *Pitesti University Publishing House*, 2013.

[4] Octavian Cira, "*Mathcad Lessons*", ISBN 973-9443-24-9, Blue Publishing House, Cluj-Napoca, 2000.

[5] Rodica-Mihaela Teodorescu, "*Synthesis-based design of RC one-ports*", ISSN 2457-7812, Journal of Electrical Engineering, Electronics, Control and Computer Science, http://jeeeccs.net, Volume 2, Number 1, May 2016.