FOURIER SERIES USING VIRTUAL INSTRUMENTATION

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Abstract - *The paper presents the Fourier series, using the virtual tool made in the Labview graphic programming environment, for the analysis and synthesis of periodic signals. Starting from the theoretical study, we arrive at the presentation of the results obtained, on the front panel, for four types of periodic signals. The paper comes in support of those who want to create a virtual tool to quickly and accurately analyze and synthesize the proposed periodic signals.*

I. INTRODUCTION

The fundamental problems that are solved with the help of Fourier series development are: *analysis and synthesis of a periodic signal.*

The analysis of a periodic signal consists in establishing the fundamental harmonic signals that compose it, and from a mathematical point of view this means determining the Fourier coefficients in the series corresponding to the function that describes the signal.

The synthesis of a periodic signal consists in establishing the combination of fundamental harmonics by superimposing on which the desired signal is obtained, and from a mathematical point of view this means the study of the convergence of the Fourier series attached to the considered fundamental harmonics.

The virtual instrument, made in the Labview graphics programming environment, allows the user to select the type of periodic signal, the minimum and maximum number of harmonics, but also the repetition period.

The interpretation of the obtained results is based on the theoretical study, regarding the determination of the Fourier series, knowing the spectrum function for the given time function.

II. FOURIER SERIES OF SIGNALS PERIODIC

A. *FOURIER FOR IMPULSE RECTANGULAR TYPE*

Rectangular pulse represented in figure 1:

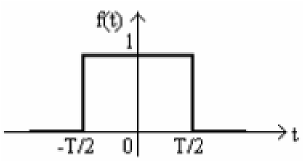

Figure 1 – Rectangular impulse

Function spectrum $F(\omega)$ function of $f(t)$ time in figure 1 is [1]:

$$F(\omega) = T \cdot \frac{\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)} = A(\omega)$$

(1)

(function spectrum can be written: $F(\omega) = A(\omega) - j \cdot B(\omega)$ and for real $F(\omega) = A(\omega)$ functions:).

The function resulting from the periodicity of the rectangular impulse, figure 2, denoted by $c(t)$, is given by the relation:

$$c(t) = a_0 + a_1 \cdot \cos(\omega_0 t) + \cdots + a_n \cdot \cos(n\omega_0 t)$$

(2)

the initial function $f(t)$ being an even function, it results that $b_n = 0$.

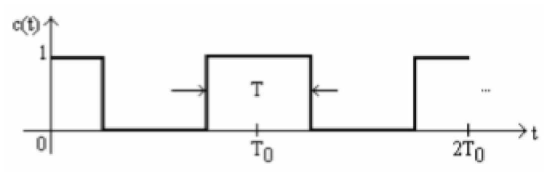

Figure 2 – Function $c(t)$ - *rectangular impulse*

The calculation relations for the Fourier series coefficients are:

$$a_0 = \frac{1}{T_0} \cdot A(0) \quad (3)$$

$$a_n = \frac{2}{T_0} \cdot A(n\omega_0) \quad (4)$$

In the particular case, of the rectangular pulse, the relations are obtained:

$$A(0) = T \quad (5)$$

$$A(n\omega_0) = T_0 \cdot \frac{\sin(n\pi r_1)}{(n\pi)} \quad (6)$$

where: $r_1 = \frac{T}{T_0}$ is the filling coefficient, T is the width of the rectangular pulse, and T_0 is the repetition period.

It follows:

$$a_0 = \frac{T}{T_0} = r_1 \quad (7)$$

$$a_n = 2 \cdot \frac{\sin(n\pi r_1)}{(n\pi)} \quad (8)$$

The Fourier series, resulting from the periodicity of the rectangular pulse, taking into account relations (2), (7) and (8) is:

$$c(t) = r_1 + \frac{2}{\pi} \cdot \Big[\sin(r_1\pi)\cos(\omega_0 t) + \frac{1}{2}\sin(2r_1\pi)\cos(2\omega_0 t) + \cdots + \frac{1}{n}\sin(nr_1\pi)\cos(n\omega_0 t) \Big] \quad (9)$$

In compact form this relation is written:

$$c(t) = r_1 + \sum_n \left[\frac{2}{\pi} \frac{\sin(n\pi r_1)}{n} \cos(n\omega_0 t) \right] \quad (10)$$

B. *FOURIER SERIES FOR THE TRIANGULAR PULSE*

Triangular pulse represented in figure 3:

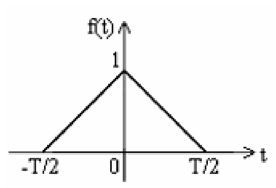

Figure 3 – *Triangular impulse*

The spectral $F(\omega)$ function of the time function $f(t)$ in figure 3 be [1]:

$$F(\omega) = \frac{T}{2} \cdot \left[\frac{\sin\left(\frac{\omega T}{4}\right)}{\left(\frac{\omega T}{4}\right)} \right]^2 = A(\omega) \quad (11)$$

The function resulting from the periodicity of the triangular pulse, figure 4, denoted by $c(t)$, is given by the relation:

$$c(t) = a_0 + a_1 \cdot \cos(\omega_0 t) + \cdots + a_n \cdot \cos(n\omega_0 t) \quad (12)$$

thefunction initial $f(t)$ being an even function, it results that: $b_n = 0$.

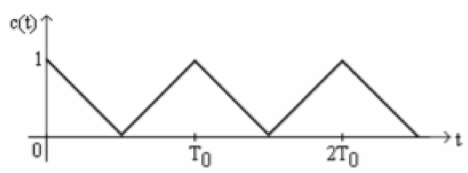

Figure 4 – *Function* $c(t)$ - *triangular impulse*

In the particular case of the triangular pulse, the relations are obtained:

$$A(0) = \frac{T}{2} \quad (13)$$

$$A(n\omega_0) = \frac{2}{r_2} \cdot T_0 \cdot \frac{\left[\sin\left(n\pi \frac{r_2}{2}\right)\right]^2}{\left[(n\pi)\right]^2} \quad (14)$$

where: $r_2 = \frac{T}{T_0}$ is the filling coefficient, T is the width of the triangular pulse, and T_0 is the repetition period.

It follows:

$$a_0 = \frac{r_2}{2} \quad (15)$$

$$a_n = \frac{4}{r_2} \cdot \frac{\left[\sin\left(n\pi \frac{r_2}{2}\right)\right]^2}{\left[(n\pi)\right]^2} \quad (16)$$

The Fourier series, resulting from the periodicity of the triangular pulse, taking into account relations (12), (15) and (16) is:

$$c(t) = \frac{1}{2} r_2 + \frac{1}{r_2} \frac{4}{\pi^2} \cdot \left[\cos(\omega_0 t) + \frac{1}{9} \cdot \cos(3\omega_0 t) + \cdots + \frac{1}{n^2} \cos(n\omega_0 t)\right] \quad (17)$$

In compact form this relation is written:

$$c(t) = \frac{1}{2} \cdot r_2 + \frac{4}{r_2 \pi^2} \cdot \sum_n \frac{1}{(2n+1)^2} \cdot \cos\left[(2n+1)\omega_0 t\right] \quad (18)$$

C. FOURIER SERIES FOR BOOST-TYPE TRAPEZOIDAL

Trapezoidal impulse represented in figure 6:

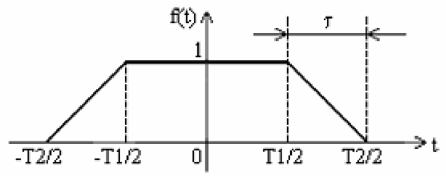

Figure 5 – Trapezoidal impulse

Function spectrum $F(\omega)$ function of timein $f(t)$ figure 5 is [1]:

$$F(\omega) = \frac{T_1 + T_2}{2} \cdot \frac{\sin\left[\frac{\omega(T_1 + T_2)}{4}\right]}{\left[\frac{\omega(T_1 + T_2)}{4}\right]} \cdot \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} \quad (19)$$

where: $\tau = \frac{T_2 - T_1}{2}$.

The function resulting from the periodicity of the trapezoidal pulse, figure 6, denoted by $c(t)$, is given by the relation:

$$c(t) = a_0 + a_1 \cdot \cos(\omega_0 t) + \cdots + a_n \cdot \cos(n\omega_0 t) \quad (20)$$

the initial function $f(t)$ being an even function, it results that: $b_n = 0$.

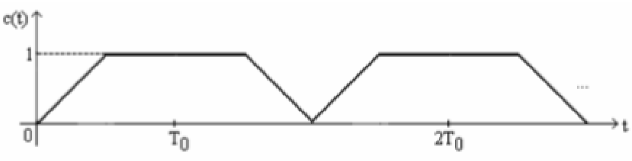

Figure 6 - Function $c(t)$ - trapezoidal impulse

In the particular case of the trapezoidal pulse, the relations are obtained:

$$A(0) = \frac{T_1 + T_2}{2} \quad (21)$$

$$A(n\omega_0) = \frac{T_0^2}{2\tau n^2 \pi^2} \cdot \left[\cos\left(\frac{n\pi T_1}{T_0}\right) - \cos\left(\frac{n\pi T_2}{T_0}\right)\right] \quad (22)$$

Follows:

$$a_0 = \frac{1}{T_0} \cdot \frac{T_1 + T_2}{2} \quad (23)$$

$$a_n = \frac{T_0}{\tau n^2 \pi^2} \left[\cos\left(\frac{n\pi T_1}{T_0}\right) - \cos\left(\frac{n\pi T_2}{T_0}\right)\right] \quad (24)$$

Fourier series, resulting from the periodicity of the pulse trapezoidal form compact,

$$c(t) = \frac{T_1 + T_2}{2T_0} + \sum_n \frac{T_0}{\tau n^2 \pi^2} \cdot \left[\cos\left(\frac{n\pi T_1}{T_0}\right) - \cos\left(\frac{n\pi T_2}{T_0}\right)\right] \cdot \cos(n\omega_0 t) \quad (25)$$

D. THE FOURIER SERIES FOR THE IMPULSE EXPONENTIAL

Whether the impulse exponentially figure 7:

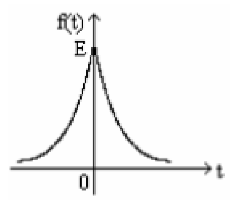

Figure 7 – Exponentially impulse

Function spectrum $F(\omega)$ of the time function $f(t)$ in figure 7, is [1]:

$$F(\omega)=\frac{2\alpha E}{\alpha^2+\omega^2}=A(\omega) \qquad (26)$$

The function resulting from the periodicity of the exponential pulse, figure 8, denoted by $c(t)$, is given by the relation:

$$c(t)=a_0+a_1\cdot\cos(\omega_0 t)+\cdots+a_n\cdot\cos(n\omega_0 t) \quad (27)$$

the initial function $f(t)$ being an even function, it results that: $b_n=0$.

In the particular case of the exponential pulse there are obtained the following relations:

$$A(0)=\frac{2E}{\alpha} \qquad (28)$$

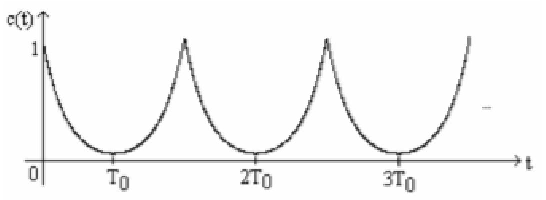

Figure 8 - Function $c(t)$ *- exponentially impulse*

$$A(n\omega_0)=\frac{2\alpha E T_0^2}{\alpha^2 T_0^2+4n^2\pi^2} \qquad (29)$$

Follows:

$$a_0=\frac{1}{T_0}\cdot\frac{2E}{\alpha} \qquad (30)$$

$$a_n=\frac{4\alpha E T_0^2}{\alpha^2 T_0^2+4n^2\pi^2} \qquad (31)$$

Fourier series, resulting in a trapezoidal pulse frequency, compact shape, is:

$$c(t)=\frac{2E}{2T_0}+\sum_n\frac{4\alpha E T_0}{\alpha^2 T_0^2+4n^2\pi^2}\cdot\cos(n\omega_0 t) \quad (32)$$

III. PRESENTATION OF COMPLETED VIRTUAL INSTRUMENT LABVIEW GRAPHICAL PROGRAMMING ENVIRONMENT

The front panel allows, through the controls, the selection of the periodic signal, the minimum and maximum number of harmonics, the repetition period of the signal, the filling factor and through the indicators the tracking of the obtained results.

A. *Presentation of the front panel for the Fourier series*

Figure 9 a) and 9 b) show two cases in which the maximum number of harmonics is different, for the rectangular pulse. Increasing the number of harmonics allows the transition from the cosine-type signal to the analyzed rectangular-type signal[2], [3], [4], [5], [6].

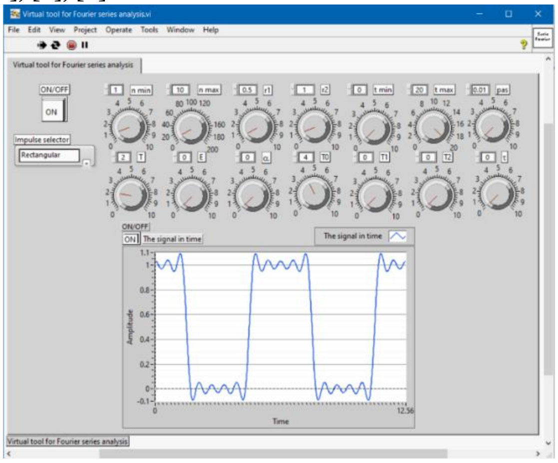

Figura 9 a) - Front panel forrectangular pulse

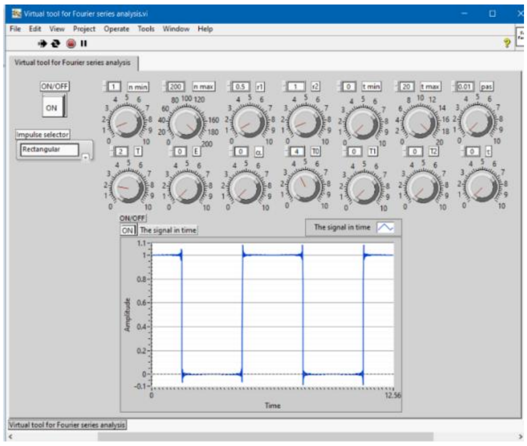

Figura 9 b) - Front panel forrectangular pulse

Figure 10 a) and 10 b) show two cases in which the maximum number of harmonics is different, for the triangular type pulse. Increasing the number of harmonics allows the transition from the cosinusoidal signal to the analyzed triangular signal[2], [3], [4], [5], [6].

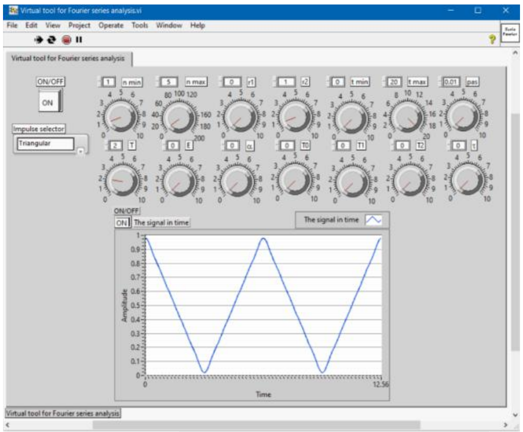

Figura 10 a) - Front panel for triangular pulse

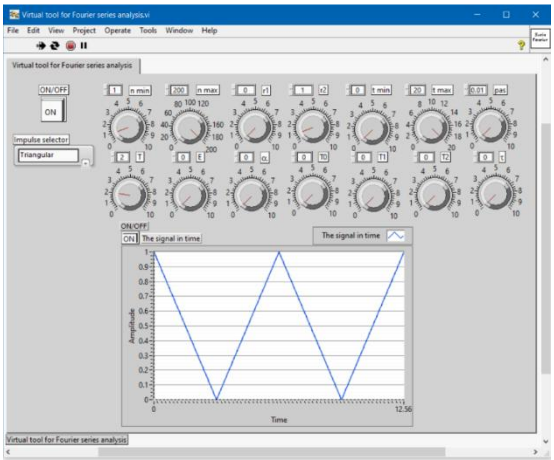

Figura 10 b) - Front panel for triangular pulse

Figure 11 a) and 11 b) show two cases in which the maximum number of harmonics is different, for the trapezoidal pulse. Increasing the number of harmonics allows the transition from the cosine type signal to the analyzed trapezoidal type signal[2], [3], [4], [5], [6].

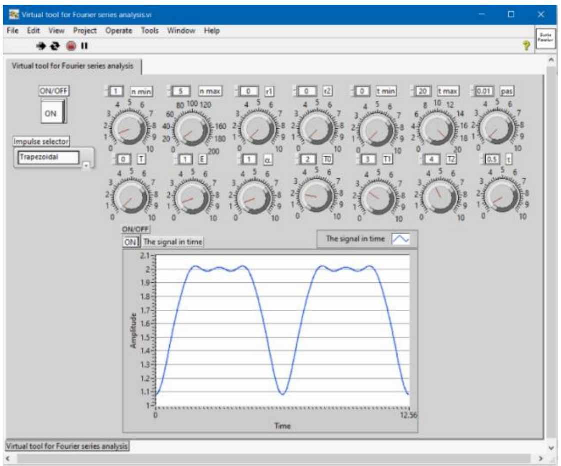

Figura 11 a) - Front panel for trapezoidal pulse

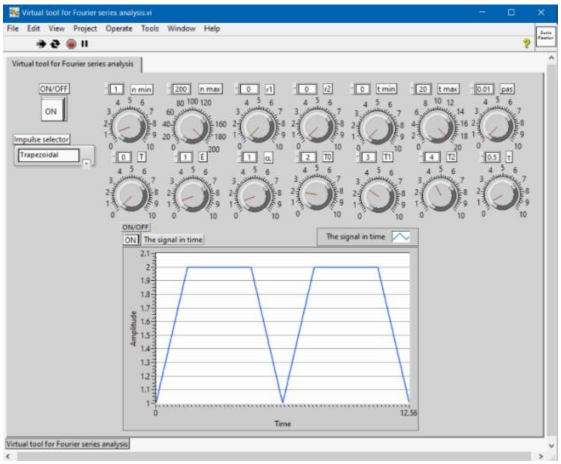

Figura 11 b) -Front panel for trapezoidal pulse

Figure 12 a) and 12 b) show two cases in which the maximum number of harmonics is different, for the exponential type pulse. Increasing the number of harmonics allows the transition from the cosinusoidal signal to the analyzed exponential signal[2], [3], [4], [5], [6].

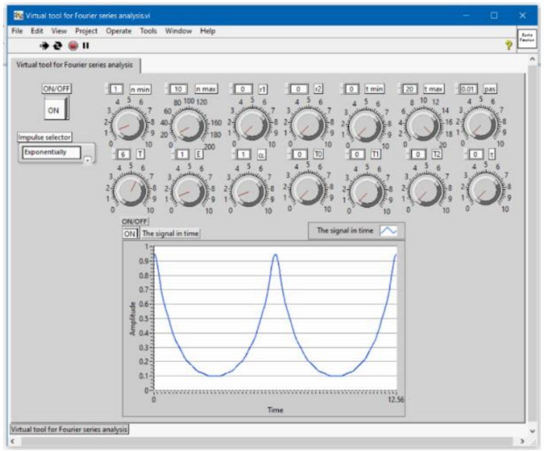

Figura 12 a) -Front panel for exponential pulse

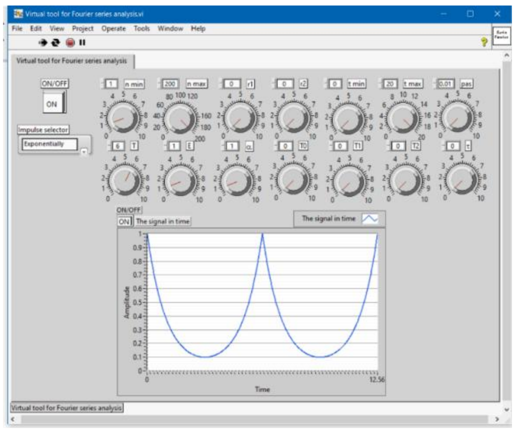

Figura 12 b) - Front panel for exponential pulse

B. Presentation of the block diagram

Figure 13 shows the block diagram of the virtual instrument for the analysis and synthesis of the Fourier series of these periodic signals [2], [3], [4], [5], [6].

The block diagram represents the program itself and contains the source code of the virtual instrument.

Each control or indicator on the front panel has a correspondent on the block diagram, which is called a terminal.

The nodes in the block diagram are objects that have inputs and / or outputs and that perform certain operations in the operation of the virtual instrument.

The wires define and graphically represent the data flow (between which nodes the information is exchanged) in the block diagram. The threads encode the type of data transmitted by the color and style of the line.

The role of the data flow is to graph the representation of the algorithm after which the virtual instrument will process the input data to calculate the output values.

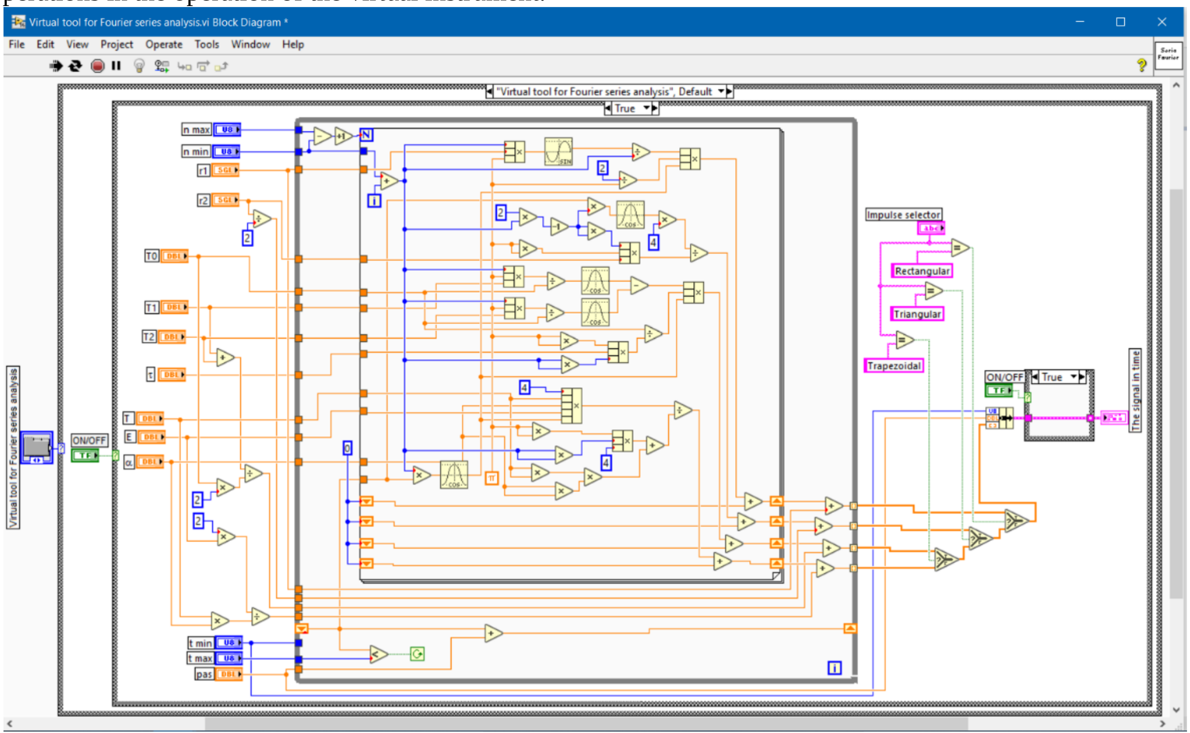

Figure 13 - Block diagram of the virtual instrument

IV. CONCLUSIONS

The paper can be useful in teaching, for those studying the analysis and synthesis of periodic signals, using the Fourier series.

For this study, those interested can make their own virtual instrument, with the possibility to select the type of periodic signal proposed for analysis, the minimum and maximum number of harmonics, the signal repetition period.

Due to the high degree of flexibility, the performance of the virtual instrument can be increased, any additional function can be easily implemented.

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