APPLICATIONS WITH VIRTUAL RESULTS IN THE STUDY OF SYSTEMS STABILITY

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Abstract: *This study is addressed especially to ones that carry out their activity in the university environment, interested to do their own virtual instrument for the study of the systems stability. The authors are presenting the study of the systems stability in three cases: stable system, unstable system, and at the limit of stability system. Due to the high grade of flexibility, the performance of the virtual instruments can increase, any additional function being able to be easily implemented.*

1. INTRODUCTION

In order to apply the fundamental stability criterion, *it is necessary to solve the characteristic equation of the system*, solving that is difficult when the order of the equation is higher than four, [1].

The denominator of the transfer function equal to zero is *the characteristic equation* of the differential equation of the given system.

For stability, the transfer function's numerator must have a degree that is less than or equal to the degree of the denominator.

The study of linear systems stability can be reduced to *the study of the geometric locus of the characteristic equation related to the imaginary axis (the pole study).*

A system is considered *stable* when the characteristic equation has roots situated to the left of the imaginary axis in the complex plane.

On the other hand, if the characteristic equation contains roots on the left side of the imaginary axis in the complex plane and also includes at least one pair of simple imaginary roots, the system is said to be *at the limit of stability*.

The system is unstable if the characteristic equation has a root on the right half of the complex plane of roots, or multiple roots on the imaginary axis.

2. METHODS FOR ANALYZING THE STABILITY OF SYSTEMS

- The Nyquist stability criterion is a frequential criterion. This allows to be estimated the stability of an automatic system in a closed circuit, based on the transfer function of the hodograph from the open loop $G(s)$. The hodograph for $G(s)$ is traced and it is analyzed the stability for:

$$H(s)=\frac{G(s)}{1+G(s)}=\frac{1}{\frac{1}{G(s)}+1}$$

For two distinct situations, the ***Nyquist stability criterion*** can be defined as, [2], [3]:

a) a system with reaction, stable with an open loop (the transfer function does not have a pole with the real positive part), is also stable with the closed reaction loop only if the hodograph of the function $G(j\omega)$, traced for $-\infty<\omega<\infty$, ***does not surround the critical point of coordinates*** $(-1, j0)$ figure 1a); From the practical point of view, for good stability, it is necessary to trace the hodograph as far as possible from the critical point $(-1, j0)$;

b) a system with reaction, unstable with an open loop (the transfer function has one or more poles with the real positive part), in order to be stable with the closed reaction loop it is only necessary for the hodograph of the open system function, when ω varies from $-\infty$ to $+\infty$, to surround k times the critical point in a trigonometric direction, respectively when ω varies from zero to $+\infty$, the hodograph of the open system to surround $2k$ times the critical point figure 1b).

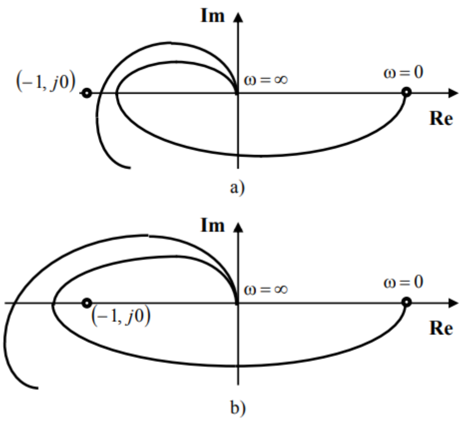

Fig. 1 The hodograph of the transfer function for a closed system: a) stable; b) unstable

- The method of the geometric locus of roots **$\mathcal{L}$**, offers the possibility to localize, in the complex plan, the characteristic equation roots of a closed loop system through negative united, when the amplification factor from the direct path K, varies on the $[0, \infty)$, [1].

The open loop tr
ansfer function has the following expression:

$$G(s) = \frac{k \cdot M(s)}{N(s)} = \frac{k \cdot \prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}$$

and for the closed loop transfer function is:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{k \cdot \prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i) + k \cdot \prod_{i=1}^{m}(s - z_i)}$$

Noting: $p(s) = \prod_{i=1}^{n}(s - p_i) + k \cdot \prod_{i=1}^{m}(s - z_i)$ the equation of the poles, it can be observed that the poles of the closed loop system depend on the parameter $k \geq 0$ and the poles and zeros of the open loop system.

There are a series of rules and an algorithm that allows tracing of the ***hodograph*** for the closed loop system poles.

Rule 1: The initial points of **$\mathcal{L}$** are the poles $p_i, i = \overline{1, n}$, of $G(s)$.

So: **$\mathcal{L}$** will have n initial points.

Rule 2: The final points of **$\mathcal{L}$** are the zeros $z_i, i = \overline{1, m}$, of $G(s)$.

So: **$\mathcal{L}$** will have m final points with a finite number.

Rule 3: If the transfer function $G(s)$ is strictly proper, then $m < n$ so there will be $n - m$ branches that will asymptotically tend to infinite. In this case, it traverses the point of intersection σ_a of the asymptotes with the real axis and with the θ_r angles, $r = \overline{0, n - m - 1}$ that these asymptotes make with the real axis.

Those will be calculated with the formulas:

$$\sigma_a = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n - m}, \quad \theta_r = \frac{(2r + 1)\pi}{n - m}$$

for $r = \overline{0, n - m - 1}$.

Rule 4: **$\mathcal{L}$** includes the line segment from the left of the pole or zero in question, only if there are an odd number of poles and real zeros to its right, counting their multiplicity and including the point itself.

Rule 5: If the line segment comes from a pole from the left side of another point, , there is a break from the real axis between these two points.The detachment point is ***the real root*** of the equation:

$$\sum_{i=1}^{n} \frac{1}{s - p_i} - \sum_{i=1}^{m} \frac{1}{s - z_i} = 0$$

that belongs to the interval between these two points.

Rule 6: The intersection **$\mathcal{L}$** with imaginary axis is obtained for $k \geq 0$ values that fulfill the automatic system found at the limit of stability.

It is determined the value that is canceled at the penultimate determinant of the Hurwitz matrix for $p(s)$ polynom.

The intersection of **$\mathcal{L}$** with imaginary axis (if there is one) is **pure imaginary** s values of the $p(s)=0$ equation after the k replacement with a determined value.

Rule 7: When there are multiple real zeros and real poles in a system, the number of incoming and outgoing roots is equal to the multiplicity of the respective zero and pole.

The directions of the tangents to the respective branches (in zeros, respectively the corresponding multiplicity poles q) are:

$$\theta_r = \frac{2\pi r}{q}, \quad r=\overline{0, q-1}$$

if the number of the real roots (poles and zeros), is situated at the right side of the considered multiply root (zero or pole), it is odd;

$$\theta_r = \frac{(2r+1)\pi}{q}, \quad r=\overline{0, q-1}$$

if the number of the real roots (poles and zeros), is situated at the right side of the considered multiply root (zero or pole), it is even.

Rule 8: It is traced the geometric locus of **$\mathcal{L}$**.

- The elementary **Bode diagrams**: (the real characteristics are with interrupted line, and the asymptotes are with continuous line), figure 2, [4].

Observation: in cases 3 and 4 of the Bode diagrams, the real characteristics have a deviation from the asymptotes: at the amplification, the deviation is 3dB, the phase shift is , but in cases 5 and 6 the real characteristics can have a higher or a less deviation depending on ξ.

Observation: in case of multiple poles (zeros), in the amplification diagram, it is multiplied the asymptote slope by the order of the pole (zero), and in the phase shift diagram it is multiplied the oblique asymptote slope by the order of the pole (zero).

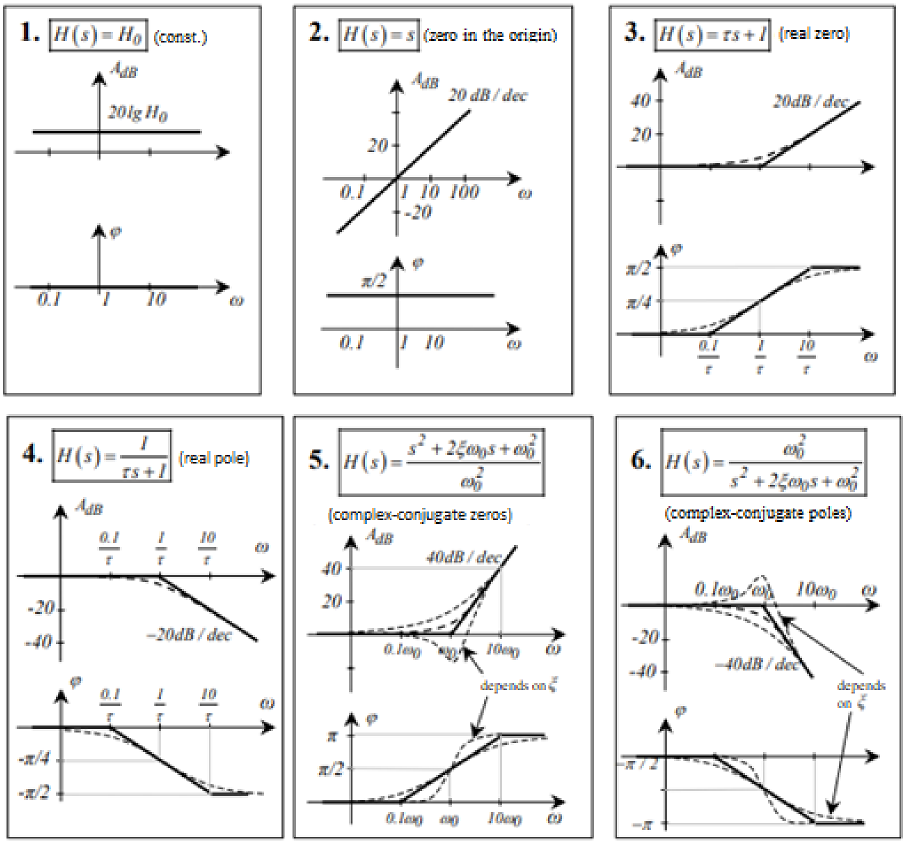

Fig. 2 Bode diagrams

3. RESULTS

CASE 1 - STABLE SYSTEM

Theoretical analysis of the system stability characterized by transfer function

The transfer function of the open loop system is:

$$G_1(s) = \frac{M(s)}{N(s)} = \frac{(s+5)(s+4)}{(s+3)(s+2)(s+1)^2} =$$

$$= \frac{s^2+9s+20}{s^4+7s^3+17s^2+17s+6}$$

The grade of $M(s)$ is $m=2$, and the grade of $N(s)$ is $n=4$.

There are two zeros, given by the relation:

$$(s+5)(s+4)=0 \Rightarrow z_1=-4;\ z_2=-5$$

and four poles, given by the relation:

$$(s+3)(s+2)(s+1)^2=0$$

$$\Rightarrow p_{1,2}=-1;\ p_3=-2;\ p_4=-3$$

The system is stable because all the poles and zeros are on the left half of the complex plane of roots.

There are ***two finite points*** of roots geometric locus (***𝓛***), because $G_1(s)$ has two zeros, ***four starting points (branches)*** of roots geometric locus (***𝓛***), because $G_1(s)$ has four poles and ***two branches*** of the roots geometric locus (***𝓛***) that asymptotically tend to infinity, because $n-m=2$.

σ_a is ***the intersection point of asymptotes with the real axis***.

$$\sigma_a = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} = 1$$

θ_r are ***the angles that these asymptotes make with the real axis***.

$$\theta_r = \frac{(2r+1)\pi}{n-m} \text{ for: } r=\overline{0,n-m-1},\ r=\overline{0,1}.$$

It is obtained:

$$\text{for: } r=0 \Rightarrow \theta_1=\frac{\pi}{2};\ \text{for: } r=1 \Rightarrow \theta_2=\frac{3\pi}{2}.$$

Observations:

- The pole or zero in question and the line segment from its left are both part of the roots geometric locus (***𝓛***), only if there is an odd total number of poles and real zeros on its right, counting their multiplicity and including the point itself.
- The zero $z_2=-5$ can not bond to a left side point, because the bonding of zero with $-\infty$ is excluded;
- The zero $z_1=-4$ can not bond to a left side point, the zero $z_2=-5$, because the number of the poles and zeros from its right side, including this zero, is an odd number (equal to 5);
- The pole $p_4=-3$ can not bond to a left side point, because the poles and zeros number from its right side, including this pole, is an even number (equal to 4);
- The pole $p_3=-2$ can not bond to a left side point, $p_4=-3$, because the poles and zeros number from its right side, including this pole, is an odd number (equal to 3);
- The poles $p_1=p_2=-1$ can not bond to a left side point, because the poles and zeros number from their right side, including these poles, is an even number (equal to 2);
- Between the poles $p_3=-2$ and $p_4=-3$ there is *a* ***detaching point from the real axis***, and between the zeros $z_1=-4$ and $z_2=-5$ there is ***a returning point on the real axis.***

The detaching point is the real root of the equation:

$$\sum_{i=1}^{n}\frac{1}{s-p_i} - \sum_{i=1}^{m}\frac{1}{s-z_i} = 0$$

there is:

$$\frac{1}{s+1}+\frac{1}{s+1}+\frac{1}{s+2}+\frac{1}{s+3}-\frac{1}{s+4}-\frac{1}{s+5}=0$$

from where it results in :

$$2s^4+32s^3+174s^2+382s+286=0$$

with the roots:

$$s_1=-1.67;\ s_2=-4.31;\ s_3=-2.73;\ s_4=-7.29$$

The only real root of the polynom:

$$2s^4+32s^3+174s^2+382s+286=0$$

that represents the detaching point from the real axis and belongs to the interval $[-2,-3]$ is: $s_3=-2.73$, and the only real root that represents the returning point to the real axis and belongs to the interval $[-4,-5]$ is: $s_2=-4.31$.

So, the detaching point from the real axis is: -2.73, and the returning point to the real axis is: -4.31.

The intersection of roots geometric locus (𝓛) with the imaginary axis is obtained for $k \geq 0$ values, corresponding to the system found at the limit of stability.

It is determined the $k \geq 0$ value for that it is annulled the last by one determinant of Hurwitz matrix of polynom $p(s)$ given by the relation:

$$p(s) = \prod_{i=1}^{n}(s - p_{i)} + k \cdot \prod_{i=1}^{m}(s - z_{i)} = 0$$

there is:

$$p(s) = (s+1)(s+1)(s+2)(s+3) + k(s+4)(s+5)$$

from that:

$$p(s) = 0 \Leftrightarrow s^4 + 7s^3 + (17+k)s^2 + \\ +(17+9k)s + (6+20k) = 0$$

The last by one determinant of Hurwitz matrix of polynom $p(s)$ is:

$$T_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} = \begin{vmatrix} 7 & 1 & 0 \\ 17+9k & 17+k & 7 \\ 0 & 6+20k & 17+9k \end{vmatrix} =$$

$$= (17+k) \cdot (17+9k) \cdot 7 - (6+20k) \cdot 7 \cdot 7 - \\ - (17+9k) \cdot (17+9k) \cdot 1$$

$$T_3 = 0 \Leftrightarrow = -18k^2 - 96k + 1440 = 0$$

$$\Rightarrow k_1 = 6.667; k_2 = -12$$

Because it is requested $k \geq 0$, it is accepted as a solution: $k_1 \cong 6.6$.

The intersection points of the geometric locus of roots (𝓛) with the imaginary axis represent pure imaginary **s** values of the equation $p(s) = 0$ after substituting the determined value of k.

$$p(s) = \prod_{i=1}^{n}(s - p_{i)} + k \cdot \prod_{i=1}^{m}(s - z_{i)} = 0$$

there is:

$$p(s) = (s+1)(s+1)(s+2)(s+3) + 6.6(s+4)(s+5)$$

from that:

$$p(s) = 0 \Leftrightarrow s^4 + 7s^3 + 23.6s^2 + 76.4s + 138 = 0$$

with roots: $s_{1,2} \cong -3.5 \pm 0.65i; \quad s_{3,4} \cong \pm 3.3i$.

The pure imaginary values of the equation $p(s) = 0$ are: $s_{3,4} \cong \pm 3.3i$.

So, in these points: $\pm 3.3i$ the geometric roots locus (𝓛) intersects the imaginary axis.

Because it is difficult to manually trace ***the Nyquist hodograph*** and ***the roots geometric locus*** (𝓛), further it is presented a virtual method for system stability analyses based on designing and implementation of a virtual device, in the graphic environment Labview.

Virtual method for analyzing the system stability characterized by transfer function

The virtual method for system stability analyses using the virtual device implemented in Labview [5], [6], [7], [8], [9], [10], [11].

The block diagram of the virtual device is presented in figure 3.

Figure 4 shows the front panel of the virtual device, which displays the open loop system function, the poles and zeros values, and the stability result of the system. The graphical indicators show: the pole and zero positions in the complex S plane, the Nyquist plot, the roots geometric locus (𝓛), and the Bode diagrams – magnitude and phase.

Observation:

The results displayed on the indicators on the front panel of the virtual instrument, as shown in Figure 4, are identical to those obtained from the theoretical analysis.

CASE 2 - UNSTABLE SYSTEM

Theoretical analysis of the system stability characterized by transfer function

The transfer function of the open loop system is:

$$G_2(s) = \frac{M(s)}{N(s)} = \frac{(2s+1)^3}{5s^4 + 4s^3 + 3s^2 + 2s + 1} = \\ = \frac{8s^3 + 12s^2 + 6s + 1}{5s^4 + 4s^3 + 3s^2 + 2s + 1}$$

The grade of $M(s)$ is $m = 3$, and the grade of $N(s)$ is $n = 4$.

There are three zeros, given by the relation:

$$(2s+1)^3 = 0 \Rightarrow z_1 = z_2 = z_3 = -0.5$$

and four poles, given by the relation:

$$5s^4 + 4s^3 + 3s^2 + 2s + 1 = 0$$

$$\Rightarrow p_{1,2} = -0.54 \pm 0.36i; p_{3,4} = 0.14 \pm 0.68i$$

There are ***three finishing finite points*** of roots geometric locus (𝓛), because $G_2(s)$ has three zeros, ***four starting points (branches)*** of roots geometric locus (𝓛), because $G_2(s)$ has four poles and ***one branch*** of the roots geometric locus (𝓛) that asymptotically tend to infinity, because $n - m = 1$.

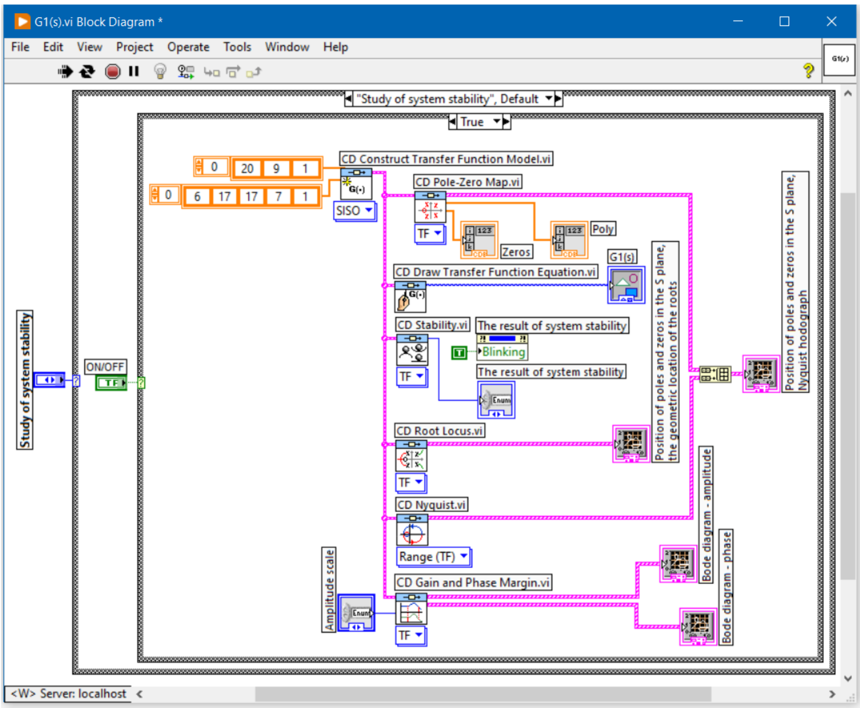

Fig. 3 Block diagram of the virtual device

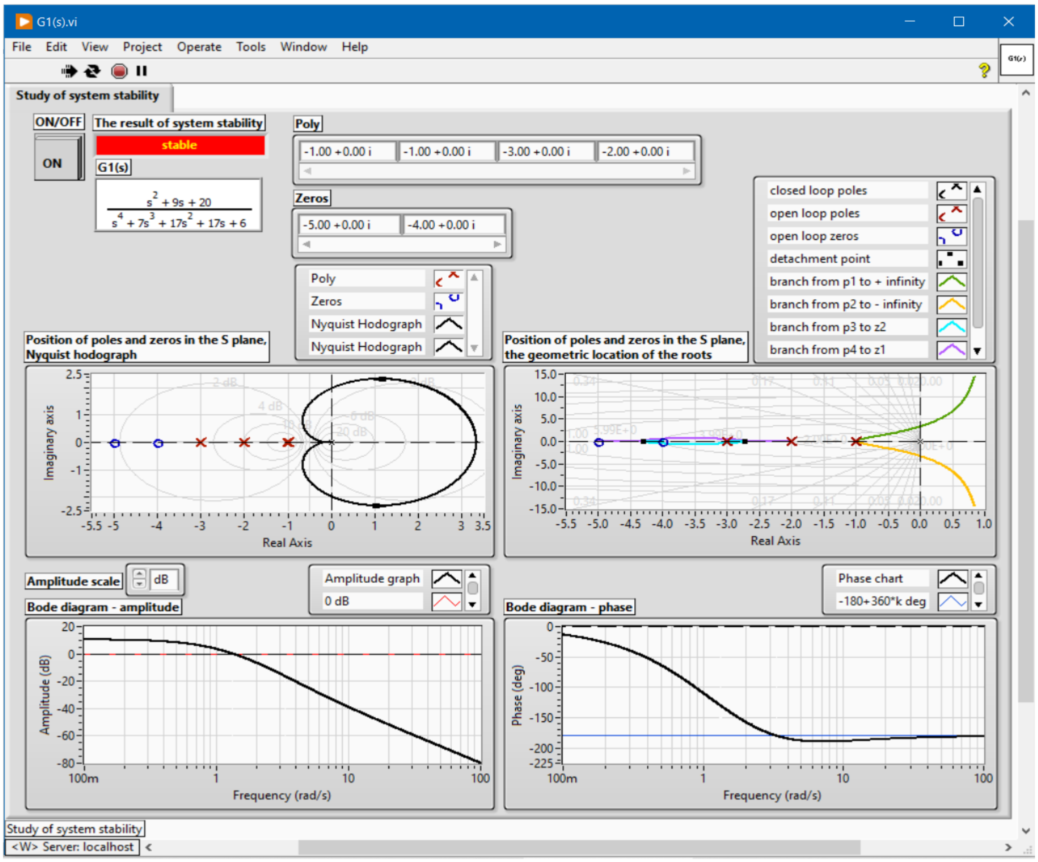

Fig. 4 The front panel of the virtual device

σ_a is ***the intersection point of asymptotes with the real axis***.

$$\sigma_a = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} = 0.7$$

θ_r are ***the angles that these asymptotes make with the real axis.***

$$\theta_r = \frac{(2r+1)\pi}{n-m} \text{ for } r = \overline{0, n-m-1},\ r = 0.$$

It is obtained for: $r = 0 \Rightarrow \theta_1 = \pi$.

Observation:

- The pole or zero in question and the line segment from its left are both part of the roots geometric locus (***ℒ***), only if there is an odd total number of poles and real zeros on its right, counting their multiplicity and including the point itself;
- The zeros $z_1 = z_2 = z_3 = -0.5$ can not bond to a left side point, because the bonding of zero with $-\infty$ is excluded;
- Between the poles $p_3 = 0.14 + 0.68i$ and $p_4 = 0.14 - 0.68i$ there is *a* ***detaching point from the real axis***.

The detaching point is the real root of the equation:

$$\sum_{i=1}^{n} \frac{1}{s-p_i} - \sum_{i=1}^{m} \frac{1}{s-z_i} = 0$$

there is:

$$\frac{1}{s+0.54-0.36i} + \frac{1}{s+0.54+0.36i} + \frac{1}{s-0.14-0.68i} + \frac{1}{s-0.14+0.68i} - \frac{1}{s+0.5} - \frac{1}{s+0.5} - \frac{1}{s+0.5} = 0$$

from where it results in that:

$$250s^4 + 500s^3 + 149s^2 - 51s - 101 = 0$$

with the roots:

$$s_1 = 0.52; s_2 = -1.65; s_{3,4} = -0.43 \pm 0.54i$$

The only real root of the polynom: $250s^4 + 500s^3 + 149s^2 - 51s - 101 = 0$ that belongs to $[-0.5, -\infty]$ is : $s_2 = -1.65$.

So, the detaching point from the real axis is: -1.65. From this point of the real axis of the geometric locus (***ℒ***) two branches are are starting in $z_3 = -0.5$ and towards $-\infty$.

The intersection of roots geometric locus (***ℒ***) with the imaginary axis is obtained for $k \geq 0$, values, corresponding to the system found at the limit of stability.

The value of $k \geq 0$ is determined such that the last but one determinant of the Hurwitz matrix of the polynomial $p(s)$ is equal to zero, as specified by the given relation:

$$p(s) = \prod_{i=1}^{n}(s - p_{i)} + k \cdot \prod_{i=1}^{m}(s - z_{i)} = 0$$

there is:

$$p(s) = (s+0.54-0.36i)(s+0.54+0.36i) \cdot \\ \cdot (s-0.14-0.68i)(s-0.14+0.68i) + \\ + k(s+0.5)(s+0.5)(s+0.5)$$

from that:

$$p(s) = 0 \Leftrightarrow \\ 5s^4 + (4+8k)s^3 + (3+12k)s^2 + \\ +(2+6k)s + (1+k) = 0$$

The last by one determinant of Hurwitz matrix of polynom $p(s)$ is:

$$T_3 = \begin{vmatrix} 4+8k & 5 & 0 \\ 2+6k & 3+12k & 4+8k \\ 0 & 1+k & 2+6k \end{vmatrix} =$$

$$= (4+8k)(3+12k)(2+6k) - \\ -(4+8k)(1+k)(4+8k) - (2+6k)(2+6k)5$$

$$T_3 = 0 \Leftrightarrow = 512k^3 + 316k^2 + 16k - 12 = 0$$

$$\Rightarrow k_1 = 0.155; \quad k_{2,3} = -0.386 \pm 0.044i$$

Because it is requested $k \geq 0$, it is accepted as a solution: $k_1 = 0.155$.

The intersection of roots geometric locus (***ℒ***) with the imaginary axis are pure imaginary **s** values of the equation $p(s) = 0$ after the replacement of the k value with the determined value.

$$p(s) = \prod_{i=1}^{n}(s - p_{i)} + k \cdot \prod_{i=1}^{m}(s - z_{i)} = 0$$

there is:

$$p(s) = (s+0.54-0.36i)(s+0.54+0.36i) \\ \cdot (s-0.14-0.68i)(s-0.14+0.68i) + \\ + 0.155(s+0.5)(s+0.5)(s+0.5)$$

from that:

$$p(s) = 0 \Leftrightarrow 5s^4 + 5.24s^3 + 4.86s^2 + 2.93s + 1.15 = 0$$

with roots:

$$s_{1,2} \cong -0.52 \pm 0.37i; s_{3,4} \cong \pm 0.75i$$

The pure imaginary values of the equation: $p(s)=0$ are: $s_{3,4} \cong \pm 0.75i$.

So, in these points $\pm 0.75i$, the geometric roots locus ($\mathcal{L}$) intersects the imaginary axis.

Because it is difficult to manually trace ***the Nyquist hodograph*** and ***the roots geometric locus*** ($\mathcal{L}$), further it is presented a virtual method for system stability analyse based on designing and implementation of a virtual device, in the graphic environment Labview.

Virtual methods for analyzing the system stability characterized by transfer function

The virtual method for analyzing system stability involves utilizing a virtual device implemented in Labview. [5], [6], [7], [8], [9], [10], [11].

The block diagram of the virtual device is presented in figure 5.

Figure 6 shows the front panel of the virtual device, which displays the open loop system function $G_2(s)$, the poles and zeros values, and the stability result of the system. The graphical indicators show: the pole and zero positions in the complex S plane, the Nyquist plot, the roots geometric locus ($\mathcal{L}$), and the Bode diagrams – phase and amplitude.

Observation:

Similarly in this case, the results displayed on the indicators on the front panel of the virtual instrument, as shown in Figure 6, are identical to those obtained from the theoretical analysis.

CASE 3 - SYSTEM AT THE LIMIT OF STABILITY

Theoretical analysis of the system stability characterized by transfer function

The transfer function of the open loop system is [3], [5], [8]:

$$G_3(s)=\frac{M(s)}{N(s)}=\frac{s^2+3s+5}{s(s+2)(s^2+2s+2)}=$$

$$=\frac{s^2+3s+5}{s^4+4s^3+6s^2+4s}$$

The grade of $M(s)$ is $m=2$, and the grade of $N(s)$ is $n=4$.

There are two zeros, given by the relation:

$$s^2+3s+5=0 \Rightarrow z_{1,2}=-1.5\pm 1.66i$$

and four poles, given by the relation:

$$s(s+2)(s^2+2s+2)=0 \Rightarrow$$

$$p_1=0;\ p_2=-2;\ p_{3,4}=-1\pm i$$

There are ***two finishing finite points*** of roots geometric locus ($\mathcal{L}$), because $G_3(s)$ has two zeros, ***four starting points (branches)*** of roots geometric locus ($\mathcal{L}$), because $G_3(s)$ has four poles and ***two branches*** of the roots geometric locus ($\mathcal{L}$) that asymptotically tend to infinity, because $n-m=2$.

σ_a is ***the intersection point of asymptotes with the real axis***.

$$\sigma_a=\frac{\sum_{i=1}^{n}p_i-\sum_{i=1}^{m}z_i}{n-m}=-0.5$$

θ_r are ***the angles that these asymptotes make with the real axis.***

$$\theta_r=\frac{(2r+1)\pi}{n-m} \text{ for: } r=\overline{0,n-m-1},\ r=\overline{0,1}$$

It is obtained:

$$\text{for: } r=0 \Rightarrow \theta_1=\frac{\pi}{2};\ \text{for: } r=1 \Rightarrow \theta_2=\frac{3\pi}{2}.$$

Observations:

- The line segment that comes from the left point of the pole belongs to the roots geometric locus ($\mathcal{L}$), respectively the discussed zero, only if in the right side of it there is an odd number of poles and ***real*** zeros considered in their totality and along with their multiplicity order, also the number that includes the point of the discussion;
- The pole $p_2=-2$ can not bond to a left side point, because the poles and zeros number from its right side, that includes this pole, is an even number (equal to 2);
- The pole $p_1=0$ bonds to a left side point, because the number of the poles and zeros from its right side, including this pole, is an odd number (equal to 1);
- Between the poles p_1 and p_2, there is *a* ***detaching point from the real axis.***

The detaching point is the real root of the equation:

$$\sum_{i=1}^{n}\frac{1}{s-p_i}-\sum_{i=1}^{m}\frac{1}{s-z_i}=0$$

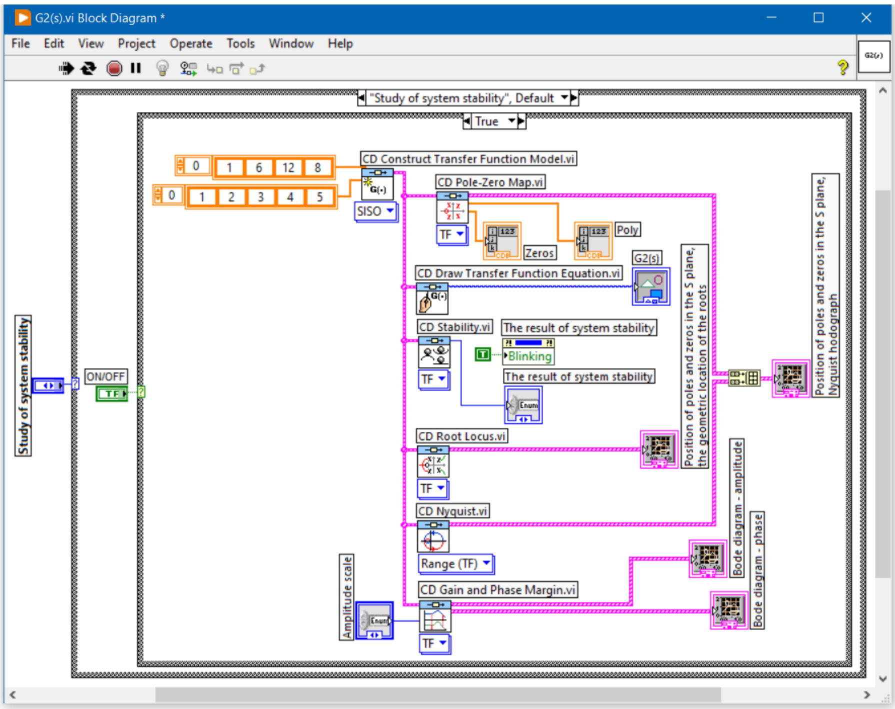

Fig. 5 Block diagram of the virtual device

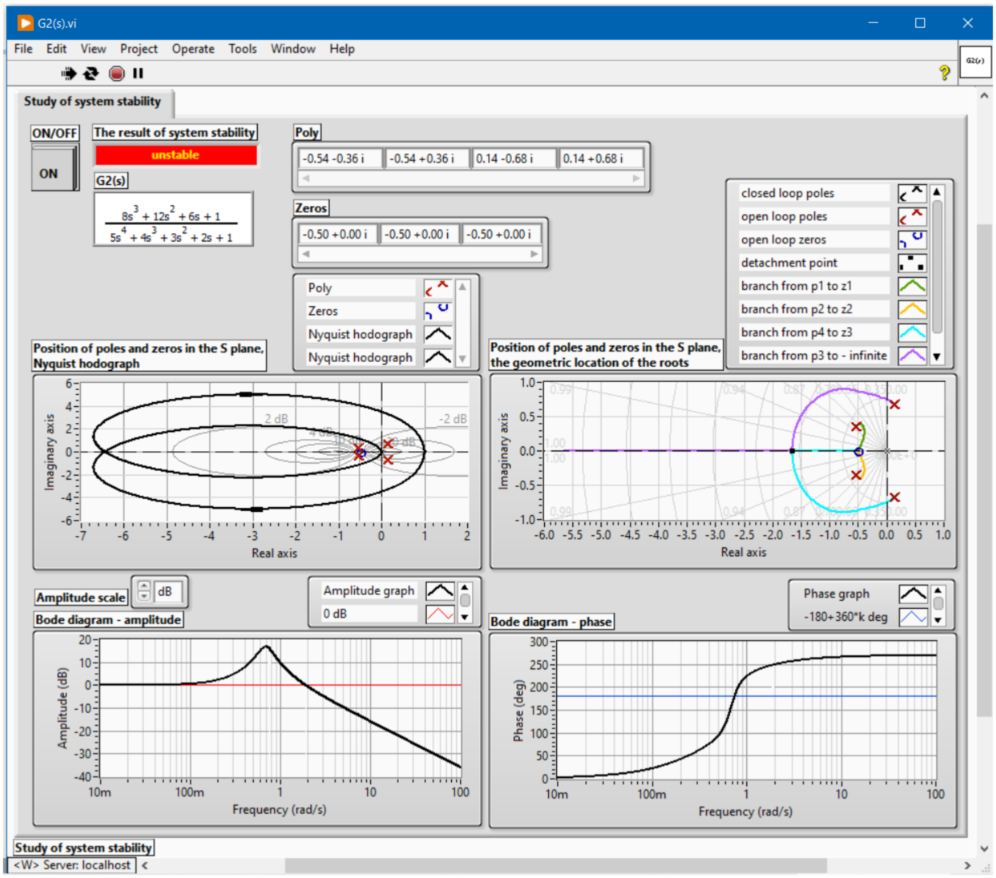

Fig. 6 The front panel of the virtual device

there is:

$$\frac{1}{s}+\frac{1}{s+2}+\frac{1}{s+1+i}+\frac{1}{s+1-i}-$$
$$-\frac{1}{s+1.5+1.66i}-\frac{1}{s+1.5-1.66i}=0$$

from where it results in:

$$25s^5+162s^4+550s^3+926s^2+751s+250=0$$

with the roots:

$$s_1=-1.31; s_{2,3}=-0.81\pm 0.48i;$$
$$s_{4,5}=-1.77\pm 2.31i$$

The only real root of the polynom:

$$25s^5+162s^4+550s^3+926s^2+751s+250=0$$

that belongs to the interval $[0,-2]$ is: $s_1=-1.31$.

So, the detaching point from the real axis is: -1.31.

The intersection of the geometric locus (**$\mathcal{L}$**) of roots with the imaginary axis is achieved for values of $k\geq 0$, indicating that the system is at the limit of stability.

It is determined the $k\geq 0$ value for that it is annulled the last by one determinant of Hurwitz matrix of polynom $p(s)$ given by the relation:

$$p(s)=\prod_{i=1}^{n}(s-p_{i)}+k\cdot\prod_{i=1}^{m}(s-z_{i)}=0$$

there is:

$$p(s)=s(s+2)(s+1+i)(s+1-i)+$$
$$+k(s+1.5+1.66i)(s+1.5-1.66i)$$

from that:

$$p(s)=0\Leftrightarrow s^4+4s^3+(6+k)s^2+(3k+4)s+5k=0$$

The last by one determinant of Hurwitz matrix of polynom $p(s)$ is:

$$T_3=\begin{vmatrix}4 & 1 & 0\\ 4+3k & 6+k & 4\\ 0 & 5k & 4+3k\end{vmatrix}=3k^2-16k+80$$

$$T_3=0\Rightarrow k_{1,2}=2.66\pm 4.42i$$

Because it is requested: $k\geq 0$ this solution is not accepted.

So, the roots geometric locus (**$\mathcal{L}$**) does not intersect with the imaginary axis, it is symmetrical to the real axis.

To make it easier to analyse the system stability, we will use a virtual device that we created in Labview. This device can show us the Nyquist plot and the roots geometric locus (**$\mathcal{L}$**) of the system, as well as other features.

Virtual method for analyzing the system stability characterized by transfer function

The virtual method for system stability analyses using the virtual device implemented in Labview [5], [6], [7], [8], [9], [10], [11].

The block diagram of the virtual device is presented in figure 7.

On the front panel of the virtual device, figure 8, it is presented the $G_3(s)$ function of the open loop system, the values for the poles and the zeros of it, the result about the stability of the system. The graphic indicators present: the pole position and for zero in the complex plan $\mathbb{C}$, the Nyquist hodograph, the roots geometric locus (**$\mathcal{L}$**), the Bode diagrams – amplitude and phase.

Observation:

Additionally, in this case, when comparing the results shown on the indicators of the front panel of the virtual instrument, as depicted in Figure 8, with those obtained from the theoretical analysis, it is observed that they are identical.

CONCLUSIONS

By comparing the results displayed on the indicators on the front panel of the virtual instrument for the three cases with those obtained from the theoretical analysis, it is evident that they are identical.

Therefore, only accurate theoretical analysis and correctly implemented virtual instruments can lead to identical results.

The virtual instrument, of the Labview programming environment can be used by anyone for the study of any system stability, when function of open loop transfer is known. It is only necessary to introduce the values of the coefficients, in block diagram of the virtual instrument, for any function of the system.

The front panel of the virtual instrument shows different aspects of the open loop system based on the coefficients values. These include: the open loop transfer function; the poles and zeros values; the poles and zeros locations in S plane; the stability result of the open loop system; the Nyquist curve; the roots geometric locus; and the Bode diagrams.

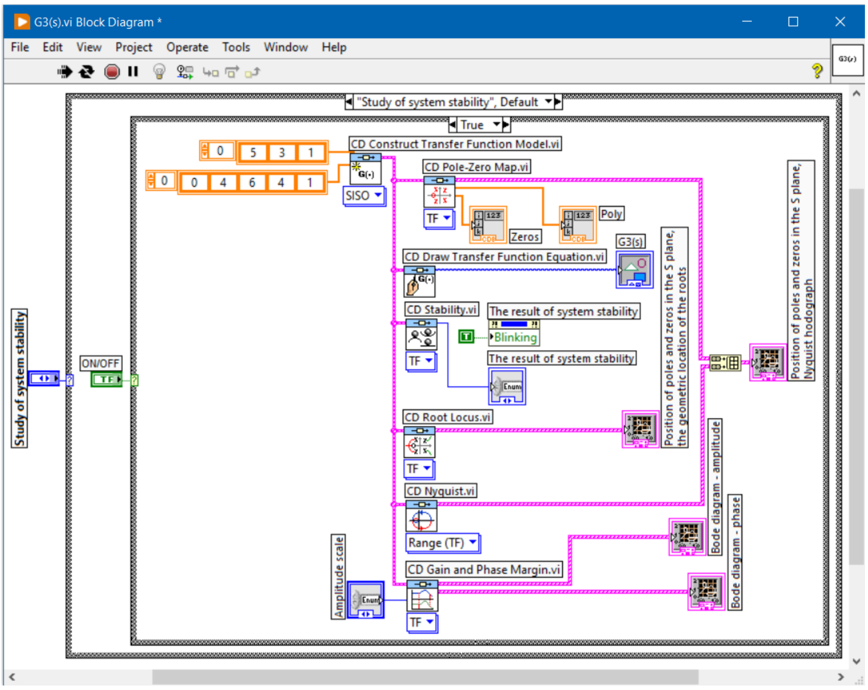

Fig. 7 Block diagram of the virtual device

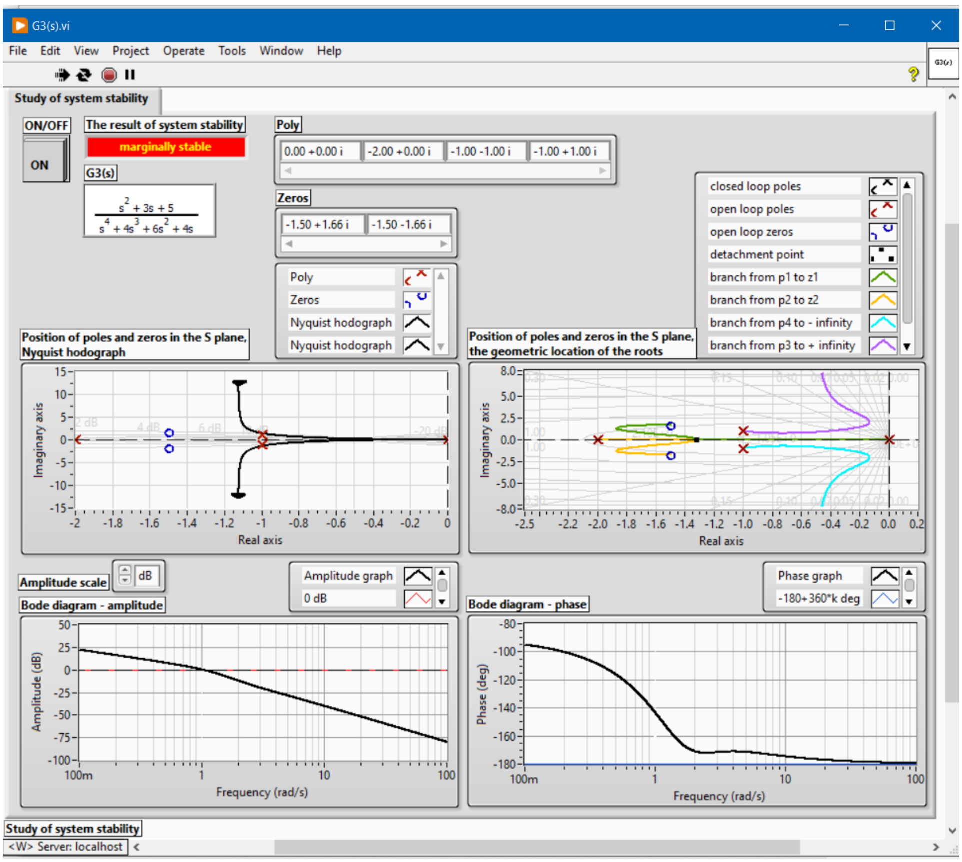

Fig. 8 The front panel of the virtual device

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